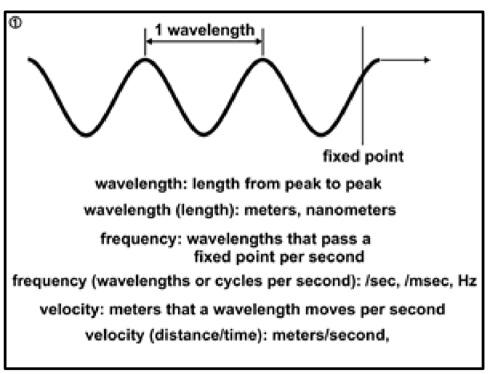
1. Match the proper units with the following:

| | |
|---|---|
| W. wavelength | 1. nm |
| F. frequency | 2. /sec |
| V. velocity | 3. m |
| | 4. $ms^{-1}$ |
| | 5. Hz |
| | 6. m/sec |

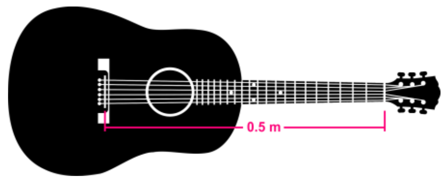
| | | |
|---|---|---|
| (A) W: 1, 3 | F: 2, 4, 5 | V: 6 |
| (B) W: 3 | F: 1, 4, 5 | V: 6 |
| (C) W: 1 | F: 2, 3, 5 | V: 6 |
| (D) W: 3, 4 | F: 2, 5, 6 | V: 4, 6 |
| (E) W: 1, 3 | F: 2, 5 | V: 4, 6 |

2. If a wave travels at 1000 meters per second, and its frequency is 40 Hz, what is its wavelength?




(A) 100 meters
(B) 75 meters
(C) 50 meters
(D) 25 meters
(E) 12.5 meters

3. A guitar string 0.5 meter long is plucked and vibrates at 290 Hz. How fast does the wave move down the guitar string?



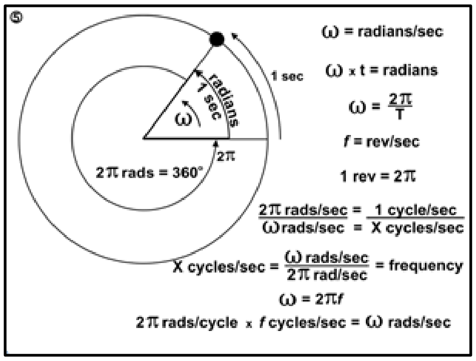

(A) 145 Hz
(B) 290 Hz
(C) 435 Hz
(D) 580 Hz
(E) 1160 Hz

4. What is the first harmonic frequency of a guitar string 0.6 meters long whose mass is $3.0 \times 10^{-4}$ kg, and pulled to a tension of 80.0 newtons?

(A) 233.3 Hz
(B) 333.3 Hz
(C) 433.3 Hz
(D) 533.3 Hz
(E) 633.3 Hz

**Introduction – Question 5**

The next five slides provide a background for harmonic motion.




For a point rotating in a circle, angular velocity ω is the number of radians the point moves in 1 second. Distance, in radians, over time is angular velocity, ω.

ω times t is the number of radians the point moves after t seconds.

ω can also be calculated by dividing the distance around the entire circle in radians, 2 π, divided by the time, capital T, that it takes to make one complete revolution.

Frequency is how many complete revolutions, or parts of a complete revolution, the point makes in 1 second. 1 complete revolution, or cycle, is 2π radians.

If, in 1 second, the point went 2π radians all the way around the circle, what fraction of the circle would the point have gone if it were traveling at ω radians per second?

If, in 1 second, 2π radians equals 1 cycle, then ω radians would equal X fraction of a cycle.

X equals ω radians traveled in 1 second divided by 2π radians.

X is the fraction of a circle, or better, the fraction of a cycle that the point travels in 1 second.

The cycles, or fraction of a cycle, that a point moves in 1 second is its frequency. And 2 π radians per cycle times the frequency in cycles per second equals ω, in radians per second.

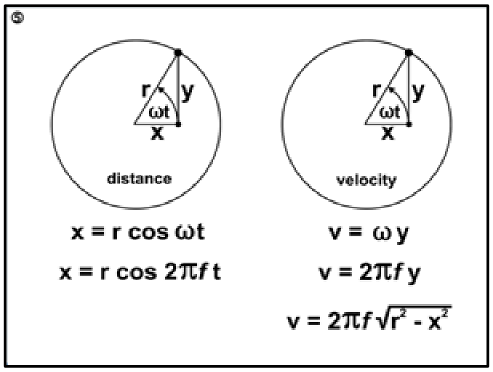
Frequency, then, is angular velocity, ω, in radians per second divided by 2π radians per cycle, which gives frequency in cycles per second.

And ω, angular frequency, is 2π times the frequency.

So if the frequency is 3 Hz, or 3 cycles per second, ω, the angular frequency, is 6 π radians per second.

There are 360 degrees in a circle, so there are 360 degrees in 2 π radians. Knowing the number of radians the point moves around the circle in 1 second allows you to readily determine the angle in degrees that it moves in 1 second.

**Introduction 2 Question 5**




The harmonic motion of a spring or a pendulum can be represented by a point moving around a circle.

The center of the circle represents the resting point of a spring, or the lowest point of a swinging pendulum.

The radius of the circle is the distance the spring is stretched, or compressed, from its resting position before being released. In other words, the radius of the circle is its maximal displacement.

Once the spring is released and it bobs back and forth, distance X represents the actual distance the spring is from its resting position.

The length of X is r times the cosine of angle ωt. R is the maximal displacement from the resting position, and ωt is the angle of rotation.

Because ω is 2 π times the spring's frequency, X equals the radius, r, times the cosine of 2 π times the frequency times t.

While the distance of the spring or pendulum from its resting point is represented by x, the velocity of the spring or pendulum is the angular velocity, ω, of the circular-moving point times the length of y.

Notice that the smaller X becomes, meaning the closer the spring gets to its resting length, the longer y becomes, and the faster the spring moves. The peak velocity of the spring and pendulum occurs when X equals zero and they are crossing their rest point, which, for a pendulum, is the bottom of its arc.

We can see this mathematically, too.

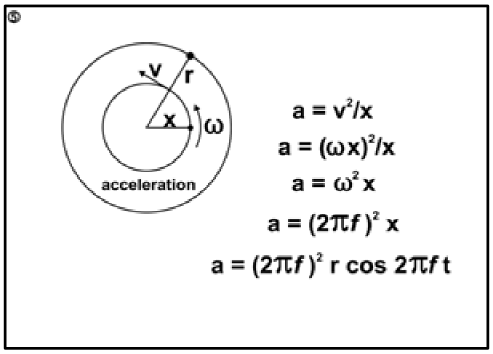
Velocity equals ω y.

ω equals 2π*f*, so velocity equals 2π*f* times y. Y, of course, being one side of a right triangle can also be expressed as the square root of r squared minus X squared. This makes the velocity of the moving horizontal point equal to ω times the square root of r squared minus X squared.

What this formula says in English is that when X equals r, that is, when the spring is displaced to its maximal length, r, r squared minus x squared equals zero, and the spring's velocity stops and the spring reverses direction.

When x equals zero, that is, when the spring crosses the resting position, the velocity of the spring is 2πfr, which is its maximal velocity.

**Introduction 3 Question 5**




The acceleration, a, of the oscillating spring at a distance X from the resting position is represented in our revolving point as the centripetal acceleration, a, of a point revolving a distance X from the central point.

Centripetal acceleration is tangential velocity squared, v squared, divided by the radius of the circle. The radius of the smaller circle is x, so the acceleration of harmonic motion is v squared over x. Centripetal acceleration is tangential velocity, v, squared over the radius of rotation, X.

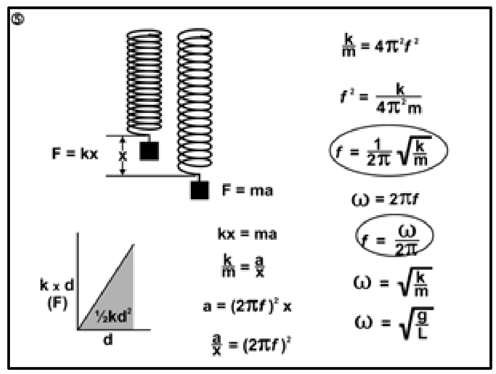
Tangential velocity, v, is angular velocity, ω, times the radius.

Centripetal acceleration, the, is ω squared times X squared over x, or ω squared times x.

Since ω is $2\pi f$, the acceleration of a harmonically moving object is $(2\pi f)^2$ times x.

Since x equals the radius of the large circle, r, times the cosine of ω t, or r times the cosine of $2\pi f t$, acceleration of a harmonically moving object is $(2\pi f)^2$ times r cos $2\pi f t$.

**Introduction 4 Question 5**




In a spring, the force needed to stretch the spring a distance x from its rest position is k, the spring constant, times x. The energy needed to stretch the spring, however, is not simply force times distance, because the force increases as the spring is stretched. Force is the spring constant, k, times the distance displaced from the resting position.

The energy exerted in stretching a spring is the area under the graph line in a force versus distance graph.

That area is one-half the spring constant times the displacement from the resting position squared.

The force stretching a spring is accelerating the mass attached to the spring according the formula force equals mass times acceleration.

This means that $kx = ma$.

Dividing each side by x and then m, we get a over x equaling k over m.

We just showed that for harmonic motion, like a bob on a spring bobbing up and down, the acceleration, a, of the bob equals x times the square of 2 π times the frequency of the harmonic motion.

Thus, for harmonic motion, a over x equals the square of 2πf.

So $4\pi^2 f^2 = \frac{k}{m}$

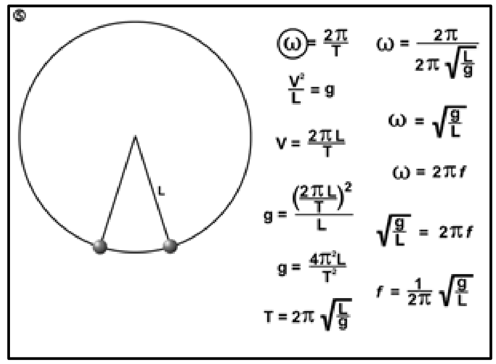
$f^2 = \frac{k}{4\pi^2}\ x$ the mass of the bob, and the frequency of harmonic motion works out to be $\frac{1}{2\pi}\ x\ \sqrt{\frac{k}{m}}$.

Because ω, angular velocity, equals 2π times the frequency, frequency equals ω divided by 2π.

When these two versions of frequency are equated, ω equals the square root of the spring constant divided by the mass of the bob.

For a pendulum, angular velocity, ω, equals the square root of the acceleration of gravity over the length of the pendulum. Here's how this angular velocity is derived.

**Introduction 5 Question 5**




Circular velocity, ω, for a pendulum is the circumference of a circle in radians, 2π, divided by the time, capital T, that it takes to make one revolution.

The centripetal acceleration for a mass moving in a circle is its tangential velocity,

V, squared, divided by the radius, which, for a pendulum, is the length of the pendulum's arm, L.

The centripetal acceleration for a pendulum is provided by the force of gravity, so the centripetal acceleration for a pendulum is the acceleration of gravity, g.

Tangential velocity is distance over time. For one complete revolution, distance is 2 π L, and T is the time it takes to make one revolution.

If g equals V squared over L, then g equals 2πL over capital T, squared, divided by L.

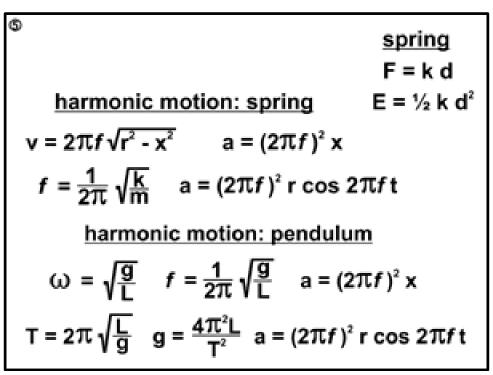
Capital T equals 2π times the square root of L over g.

Substituting this value of T into the previous equation for ω, we get ω equaling the square root of g over L.

Since ω for harmonic motion is 2 π times the frequency, the square root of g over L equals 2 π times the frequency, and the frequency of a pendulum is 1 over 2 π times the square root of g over L.

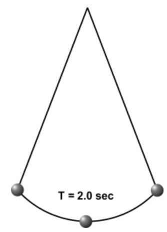
**Introduction 6 Question 5**

Here is a summary of formulas the force and energy exerted in stretching or compressing a spring, and the velocity, acceleration, frequency, and period of harmonic motion.

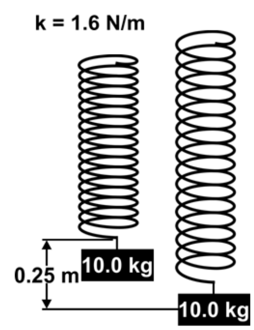



5. What is the length of a pendulum if its period of oscillation is 2 seconds?

(A) 0.5 meter
(B) 0.75 meter
(C) 1.0 meter
(D) 1.25 meter
(E) 1.5 meter



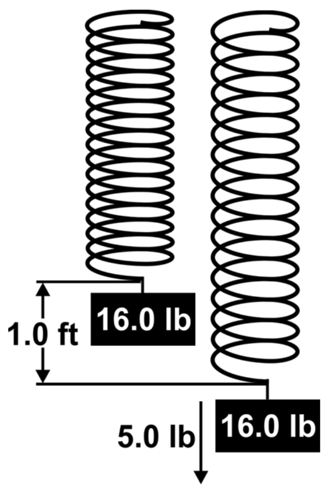

6. A 10 kg mass is suspended from a spring with a spring constant of 1.6 newtons per meter. How much would the mass weigh if placed on a scale when the spring was stretched 0.25 meters from its rest point?




(A) 95.1 N
(B) 95.4 N
(C) 96.7 N
(D) 97.0 N
(E) 97.6 N

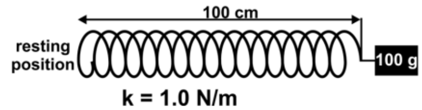
7. A 16 pound bob hanging from a spring is pulled downward 1.0 foot with a force of 5 lbs. What is the frequency of vibration after the spring is released?

When the bob springs back and reaches 3 inches above its resting position, what is its velocity and acceleration?




| Frequency | Velocity | Acceleration |
|---|---|---|
| (A) 0.25/sec | 2.4 ft/sec | 1.8 ft/sec$^2$ |
| (B) 0.5/sec | 3.0 ft/sec | 2.5 ft/sec$^2$ |
| (C) 0.6/sec | 2.4 ft/sec | 1.8 ft/sec$^2$ |
| (D) 0.8/sec | 3.0 ft/sec | 2.5 ft/sec$^2$ |
| (E) 0.9/sec | 2.7 ft/sec | 2.1 ft/sec$^2$ |

8. This spring, with a resting length of 100cm and a spring constant of 1.0 newton per meter, has a 100 gram mass attached to it. When the spring is compressed by 30 cm and released, what is the velocity of the mass when the spring reaches 100 cm and 120 cm in length?




(A) 100 cm: $\sqrt{\mathbf{0.9}\frac{\boldsymbol{m}}{\boldsymbol{sec}}}$, 120 cm: $\sqrt{\mathbf{0.5}\frac{\boldsymbol{m}}{\boldsymbol{sec}}}$

(B) 100 cm: 0.9 m/sec, 120 cm: $\sqrt{0.5\frac{\mathrm{m}}{\mathrm{sec}}}$

(C) 100 cm: $\sqrt{0.5\frac{\mathrm{m}}{\mathrm{sec}}}$, 120 cm: $\sqrt{0.9\frac{m}{sec}}$

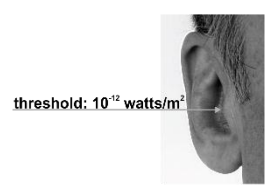
(D) 100 cm: $\sqrt{0.9\frac{m}{sec}}$, 120 cm: 0.5 m/sec

(E) 100 cm: 0.9 $\frac{m}{sec}$, 120 cm: 0.5 $\frac{m}{sec}$

9. On the moon, a 1.0 m pendulum takes 24.5 seconds to swing back and forth 5 times. What is the acceleration of gravity on the moon?

(A) 4.9 m/sec$^2$
(B) 2.4 m/sec$^2$
(C) 1.6 m/sec$^2$
(D) 1.0 m/sec$^2$
(E) 0.9 m/sec$^2$

10. The threshold for hearing is a sound intensity of $10^{-12}$ watts/m$^2$. If a sound has an intensity of $1 \times 10^{-3}$ watts/m$^2$, how many decibels is it?




(A) 3 decibels
(B) 9 decibels
(C) 12 decibels
(D) 90 decibels
(E) 120 decibels

11. How much louder in decibels is a sound with an intensity of 1 x $10^{-4}$ W/$m^2$ than a sound with an intensity of 1 x $10^{-9}$ W/$m^2$?

(A) $10^{-6}$
(B) 10
(C) $10^3$
(D) $10^5$
(E) $10^9$

12. How many times louder is a 116 db sound than a 110 dB sound?

(A) 2.22
(B) 2.89
(C) 3.10
(D) 3.98
(E) 5.62

13. How much more intense is a 150 dB sound than a 130 dB sound?

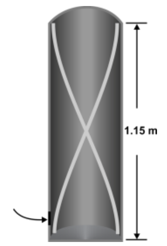
(A) 2
(B) 20
(C) 40
(D) 100
(E) 200

14. If two sounds vary in pitch, which of the following must be different: frequency, speed, amplitude, and wavelength?

(A) frequency, wavelength
(B) frequency, speed
(C) speed, amplitude
(D) amplitude, wavelength
(E) speed, wavelength

15. The first harmonic is also known as the fundamental frequency. What is the fundamental frequency of an open pipe of length 1.15 meters when the speed of sound in air is 344 meters per second?

(A) 100
(B) 125
(C) 150
(D) 175
(E) 200




16. What is the fundamental frequency of a closed pipe of length 1.15 meters when the speed of sound in air is 344 meters per second?

(A) 100
(B) 125
(C) 150
(D) 175
(E) 200