

1. How efficient is an electric motor that is able to lift a 45 kg block 16 meters upward in 15 seconds if the motor uses 4.2 A on a 120 V line?

- (A) 60%
- (B) 65%
- (C) 70%**
- (D) 75%
- (E) 80%

Efficiency is defined as the power out over the power in.

In this problem, the power out is the energy exerted per second used to lift a 40 kg mass 16 m upward against gravity. Work energy is joules, meaning force times distance.

The force needed to lift a 45 kg mass is 45 kg times the acceleration of gravity,  $9.8 \frac{m}{sec^2}$ , or  $441 \text{ kg} \cdot \frac{m}{sec^2}$ , which is 441 newtons.

When that force 441 newtons is exerted over a distance of 16 m, 441 times 16, or 7056 N-m of energy is exerted.

When these 7056 N-m, or joules, of energy are exerted over 20 seconds, the power exerted is 7056 joules over 20 seconds, or 353 joules per second, which is 353 watts.

The power in is supplied by the electric motor. Electrical power is current times voltage.

$$P = IV$$

The power in is 4.2 A times 120 V, or 504 watts.

The efficiency, then, is 353/504, or 70%.

Alternating current runs at 120 volts while a direct current car battery runs at 12 volts. To get 120 watts of power, alternating current only needs 1 amp of current, while the car battery need ten times as much current, 10 amps, which requires very thick wires. In Europe, where household voltage is 240 volts, only 0.5 amps of current is needed to generate 120 watts of power.

①

$$\text{efficiency} = \frac{\text{power out}}{\text{power in}}$$

$$\text{power} = \frac{\text{energy}}{\text{second}} = \frac{\text{joules}}{\text{second}} = \frac{\text{N} \cdot \text{m}}{\text{second}}$$

$$45 \text{ kg} \times 9.8 \text{ m/sec}^2 = 441 \text{ kg} \cdot \text{m/sec}^2 = 441 \text{ N}$$

$$441 \text{ N} \times 16 \text{ m} = 7056 \text{ kg} \cdot \text{m/sec}^2$$

$$\frac{7056 \text{ kg} \cdot \text{m/sec}^2}{20 \text{ seconds}} = 353 \frac{\text{joules}}{\text{second}} = 353 \text{ watts}$$

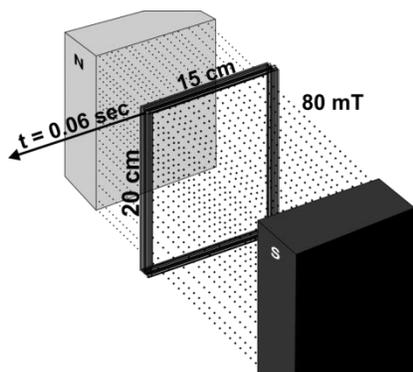
$$\text{power} = \text{current} \times \text{voltage} = IV$$

$$\text{power in} = 4.2 \text{ A} \times 120 \text{ V} = 504 \text{ watts}$$

$$\frac{353 \text{ watts}}{504 \text{ watts}} = 70\%$$

120 watts = 1 A x 120 V (alternating current U.S.)  
 120 watts = 10 A x 12 V (car battery)  
 120 watts = 0.5 A x 240 V (Europe)

2. 40 loops of wire shaped into a rectangle 15 cm wide and 20 cm long are sitting perpendicularly in a magnetic field of 80 milli Teslas (mT). What is the induced voltage if the wire loops are yanked out of the magnetic field over 0.06 seconds?



- (A) 0.4 V
- (B) 0.8 V
- (C) 1.2 V
- (D) 1.6 V**
- (E) 2.0 V

The voltage induced in a loop of wire is the rate of change of the flux passing through that loop of wire.

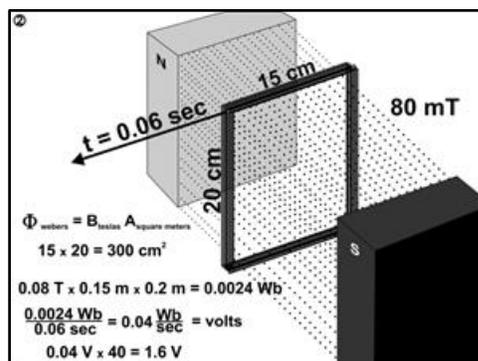
The Webers of flux passing through each loop is the magnetic field of strength 80 mT times the area of each loop, 15 by 20 cm, or 300 square centimeters.

$$0.08 \text{ T} \times 0.15 \text{ m} \times 0.2 \text{ m} = 0.0024 \text{ Wb}$$

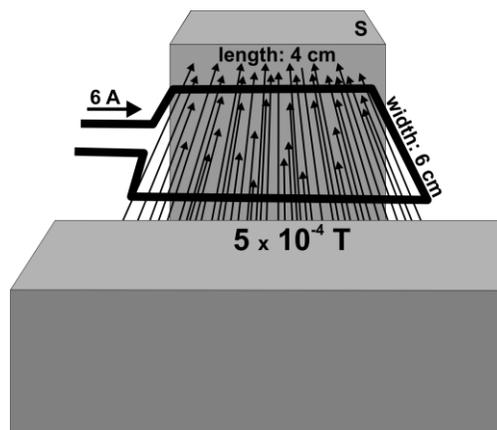
Since the 0.0024 Webers of flux dropped to zero over 0.06 seconds, the rate of the drop is

$$0.0024 \text{ Wb} / 0.06 \text{ sec} = 0.04 \text{ Wb/sec}$$

Change in flux per second is voltage, so each loop experienced a voltage of 0.04 volts, and all 40 loops experienced a total of 1.6 volts.



3. A 4 cm long by 6 cm wide rectangular loop of wire is situated in the plane of a  $5 \times 10^{-4}$  tesla magnetic field. What is the magnitude of the torque experienced by the wire loop when a 6 A current begins flowing in the wire loop?



- (A)  $2.8 \times 10^{-6}$  N-m
- (B)  $4.1 \times 10^{-6}$  N-m
- (C)  $6.6 \times 10^{-6}$  N-m
- (D)  $7.2 \times 10^{-6}$  N-m**
- (E)  $9.4 \times 10^{-6}$  N-m

The right hand rule indicates that the far wire experiences an upward force while the near wire experiences a downward force, and the wire loops rotates clockwise.

Torque is force times distance to the axis of rotation. The torque for the far wire is its upward force times 0.03 meters, which is the distance to the axis of rotation. The torque for the near wire is its downward force times also times its distance to the axis of rotation, 0.03 m.

Since the upward and the downward force each exert an equal torque, the total torque is twice the torque on each wire.

The magnetic force on each wire is the magnetic field strength,  $5 \times 10^{-4}$  teslas, times the pole strength of the electric current,  $I$  times  $l$ .  $I$  times  $l$  is 6 amps  $\times$  0.04 meters, the length of wire perpendicular to the magnetic field.

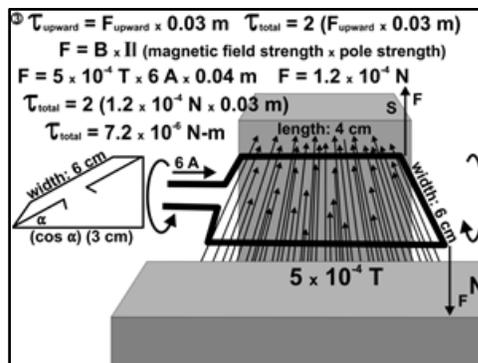
Each wire experiences a force of  $1.2 \times 10^{-4}$  newtons.

The torque on each wire is  $1.2 \times 10^{-4}$  newtons times the distance to the axis of rotation, 0.03 m, and the total torque is twice this.

The total torque is 7.2 times ten to the minus 6 newton-meters.

As the wire loop rotates, of course, the torque lessens, because the distance between each side of the wire loop and the axis of rotation lessens.

For each wire, the distance to the axis of rotation for angle alpha is half the width of the wire loop times the cosine of angle alpha (A)



#### Introduction to Question 4

A transformer is a device that allows us to increase or decrease the voltage of alternating current.

On one side of an iron ring is wound the primary coil and on the other side, the secondary coil. Either side can be considered the primary or the secondary coil.

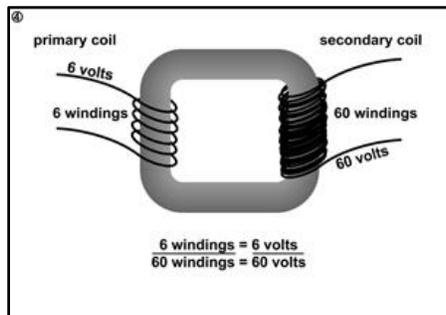
The idea in a transformer is to induce a changing magnetic field in the iron ring that matches the electrical changes in each cycle of the alternating current.

The changing magnetic field is felt throughout the iron ring. The wires in the secondary coil sense the rapidly changing magnetic field and respond with an alternating current of their own.

There is plenty of magnetic field in the iron core to supply every wire in the secondary coil, so no matter how many wires there are in the secondary coil, each wire in the secondary coil will develop the same voltage as every other wire, and together they will produce one single combined voltage in the outgoing wire.

The voltage of the secondary circuit simply depends on the number of windings in the primary and secondary coils.

So if these 6 windings in the primary coil are able to deliver 6 volts, the 60 windings in the secondary circuit will carry away 60 volts.



A transformer changes the voltage entering the transformer. It does not change the total power, however. The power leaving the transformer is the same amount of power that enters the transformer.

4. If a transformer has 400 windings in the primary circuit and 3200 windings in the secondary circuit, and the voltage in the primary circuit is 120 volts with a current of 240 mA. What is the voltage and current in the secondary circuit?

- (A) 750 V, 20 mA
- (B) 880 V, 25 mA
- (C) 960 V, 30 mA**
- (D) 1025 V, 30 mA
- (E) 1200 V, 25 mA

The ratio of the windings in the primary and secondary circuits correlates directly with voltage. Meaning, that if there are 8 times as many windings in the secondary circuit as there are in the primary circuit, the voltage in the secondary circuit is also 8 times greater, or 960 volts.

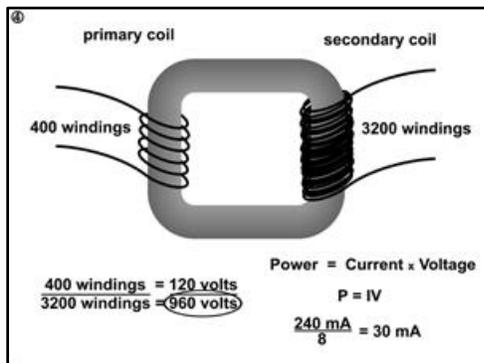
What about the current? How can we calculate the current if we don't know the resistance in the wires of the secondary current?

What we do know is that the amount of energy entering the coil has to be the same amount of energy leaving the coil. Energy per second is power, and the formula for power is current times voltage.

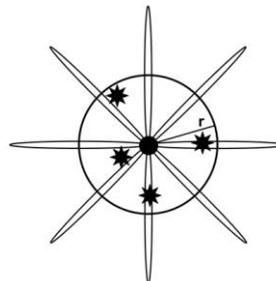
$$P = IV$$

Since current and voltage are inversely related, if the voltage is 8 times greater, the current is 8 times less.

240 mA divided by 8 is 30 m(A)



From an overhead view, you can see that the center of the circle is now experiencing the magnetic field around every point along the wire.



**Introduction to Question 5**

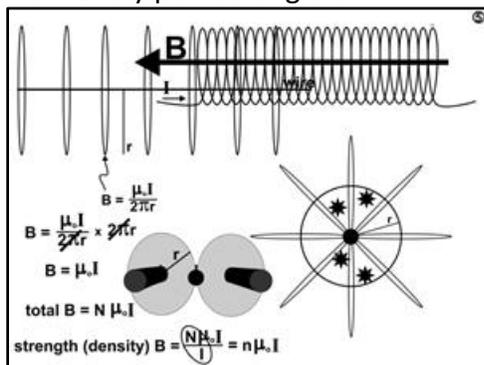
The magnetic field,  $B$ , around a straight wire carrying a current,  $I$ , is  $\mu_0$ , the permeability of free space, times the current,  $I$ , in amps, divided by the circumference around the wire,  $2\pi r$ .

The length of the wire is unimportant. At any point a distance  $r$  from the wire, the magnetic field is the same:  $\mu_0 I / 2\pi r$ .

If we now bend the wire into a circle with a radius of  $r$  and circumference of  $2\pi r$ , we end up with the center of the circle a distance  $r$  from every point along the wire.

The total magnetic field for the center of a circular wire carrying a current becomes the magnetic field for one point along the wire,  $\mu_0 I_{\text{amps}} / 2\pi r$ , times the length of the wire,  $2\pi r$ .

The magnetic field inside a single wire loop is  $\mu_0 I$ . because the radius cancels out, the radius of the wire loop is unimportant. The magnetic field is the same everywhere inside the wire loop.



Each additional wire loop adds another  $\mu_0 I_{\text{amps}}$  of magnetic field. If capital  $N$  symbolized the number of wire loops, the total magnetic field is  $N \mu_0 I_{\text{amps}}$ . A series of wire loops is called a solenoid.

The strength of the magnetic field inside the wire loops, though, is the total magnetic field divided by the length of the magnetic field.

Little  $n$ , which is capital  $N$  over  $l$  is the number of wire loops per meter.

The magnetic field, B, equals little n times  $\mu_0$  times I.

Because the magnitude of the magnetic field is the same everywhere inside each wire loop, the magnitude of the magnetic field is the same everywhere inside the solenoid and the same anywhere along the solenoid.

5. What is the magnitude and direction of the magnetic field inside this solenoid consisting of 25 loops per centimeter and carrying a current of 5.4 amps?

- (A) 0.011 T to the right
- (B) 0.011 T to the left
- (C) 0.013 T to the right
- (D) 0.017 T to the left**
- (E) 0.017 T to the right

The magnitude of the magnetic field inside a solenoid is given by the formula:

B equals  $\mu_0$ , little n times I

Little n is the number of windings per meter.

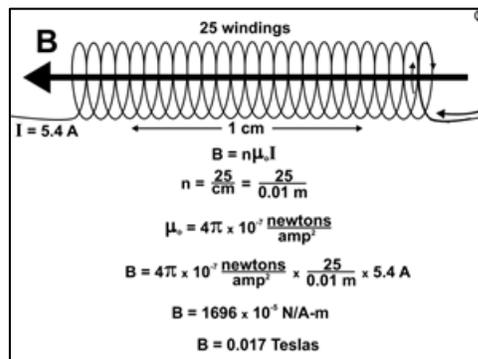
The permeability of free space is  $4\pi \times 10^{-7}$  N/A<sup>2</sup>

$$B = 4\pi \times 10^{-7} \text{ N/A}^2 \times 25 \text{ windings}/.01 \text{ m} \times 5.4 \text{ A}$$

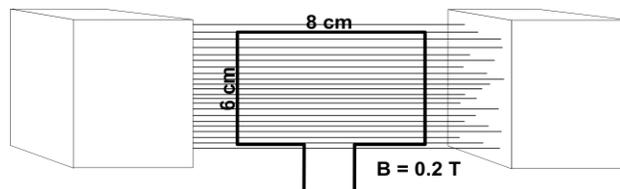
$$B = 1696 \times 10^{-5} \text{ N/A-m}$$

$$B = 0.017 \text{ T}$$

For a solenoid with the current going clockwise, the magnetic field, according to the right hand rule, points to the left.



6. A rectangular wire loop 6 cm by 8cm is rotating at a rate of 60 Hz in a magnetic field of 0.2 Telsas. What is the maximal voltage generated by the wire loop?



- (A) 0.18 V
- (B) 0.24 V
- (C) 0.36 V**
- (D) 0.48 V
- (E) 0.60 V

To review, magnetic field strength, B, is measured in Teslas.

Magnetic field strength times the surface area, A, through which the magnetic field vectors pass is the total flux, theta, measured in Webers.

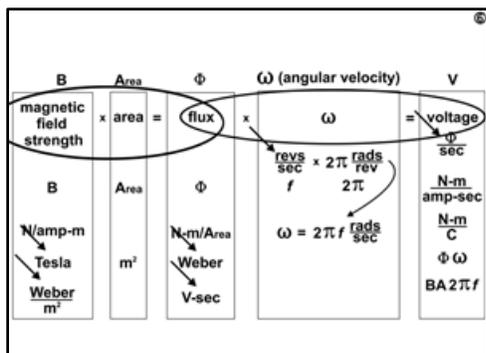
The amount of flux that changes per second is the voltage.

For wire loops that revolve in a circle, time is measured in the number of radians revolved per second, measured by angular velocity  $\omega$ . The number of radians revolved per second is the number of revolutions the object revolves per second, symbolized by the letter  $f$ , times  $2\pi$  radians per revolution.  $\omega$  equals  $2\pi f$  radians per second.

Voltage, then, equals flux times angular velocity,  $\omega$ , or magnetic field strength  $B$  times area  $A$  times angular velocity  $\omega$ , which is  $2\pi$  times frequency.

Working backward volts divided by  $\omega$  is flux, so Webers, the units for flux, are also volt-seconds.

Likewise, Webers of flux per unit area is magnetic field strength in Webers per meter squared.



6. Cont.

Maximal voltage occurs when the wire loop crosses the maximal number of magnetic field lines, which occurs when the wire loop is perpendicular to the magnetic field lines. In that case, B's magnetic field lines of 0.2 Teslas pass through the full area of the wire loop, A.

At 60 cycles per second, the wire loop travels through 60 cycles per second times  $2\pi$  radians per cycle.

Voltage, then, equals  $\omega$ , 60 cycles per second, times  $2\pi$  radians per second times B times A.

The voltage for this problem works out to be:

$$V = 60 \text{ revolutions/second} \times 2\pi \text{ radians/revolution} \times 0.2 \text{ T} \times 0.06 \text{ m} \times 0.08 \text{ m}$$

$$1V = 0.36 \text{ N-m/amp-sec}$$

Newton-meters per amp-seconds are volts because volts are a measure of work per coulomb. Work is newton-meters and coulombs are amp-seconds.

$$V = B \text{ (magnetic field strength)} \times A \text{ (area)} \times \omega \text{ (angular velocity)}$$

$$V = 0.2 \frac{\text{N}}{\text{amp-m}} \times 0.0048 \text{ m}^2 \times 376.8 \frac{\text{rads}}{\text{sec}}$$

$$V = 0.36 \frac{\text{N-m}}{\text{amp-sec}} \left( \frac{\text{work}}{\text{coulomb}} \right)$$

**Introduction to Question 7**

When the current in a solenoid increases, it induces a changing magnetic field inside the solenoid. That changing magnetic field induces its own back voltage, V, which slows the rise in current.

The magnitude of the back voltage depends on how fast the current increases – the amps per second. The back voltage induced by an increase of 1 amp per second is called the inductance of the solenoid, L. L equals V divided by 1 amp per sec, which, when divided, is volt-seconds per amp, or henries.

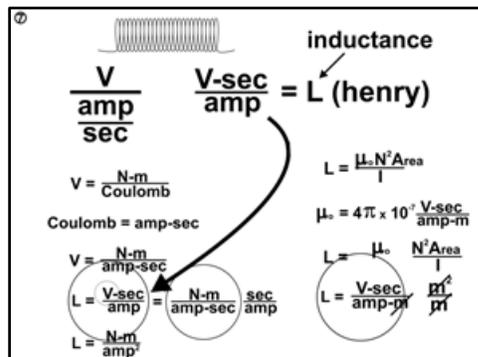
Since volts are newton-meters per coulomb, or newton-meters per amp-second, inductance, L, is also newton-meters per amp<sup>2</sup>.

Each solenoid has a specific inductance which remains constant for that solenoid.

The formula for inductance of a solenoid is the permittivity of free space,  $\mu_0$ , times the number of wire loops, capital N, squared, times the cross sectional area of the solenoid, divided by the length of the solenoid, little l.

The permittivity of free space is 4 times 10 to the minus 7 volt-sec per amp-meter.

Multiplying volt-seconds per amp-meter times area over length, the meters cancel out and the units match for inductance: volt-seconds per amp.



7. What is the inductance of a 100 winding solenoid, 10 cm long with a cross-sectional area of 40 cm<sup>2</sup>?

- (A) 5.0 x 10<sup>-2</sup> V-sec/Amp (H)
- (B) 1.0 x 10<sup>-3</sup> V-sec/Amp (H)
- (C) 2.5 x 10<sup>-4</sup> V-sec/Amp (H)
- (D) 5.0 x 10<sup>-4</sup> V-sec/Amp (H)**
- (E) 7.5 x 10<sup>-4</sup> V-sec/Amp (H)

$$L = (4\pi \times 10^{-7} \text{ V-sec/Amp-m}) (100^2) (40 \text{ cm}^2 / 0.1 \text{ m})$$

To convert square centimeters to square meters, we know that 100cm is a meter, so 100 cm times 100cm equals 1 meter squared.

$$10,000 \text{ cm}^2 = 1 \text{ m}^2$$

$$40 \text{ cm}^2 \times \text{m}^2$$

$$X = 40 / 10,000 \text{ m}^2 = 4 \times 10^{-3} \text{ m}^2$$

$$L = (4\pi \times 10^{-7} \text{ V-sec/Amp-m}) (10^4) (4 \times 10^{-3} \text{ m}^2 / 0.1 \text{ m})$$

$$L = 50.24 \times 10^{-5} \text{ V-sec/Amp}$$

$L = 5.0 \times 10^{-4} \text{ V-sec/Amp}$ , or  $5.0 \times 10^{-4} \text{ Henries}$

$$L = \frac{\mu_0 N^2 A_{\text{area}}}{l}$$

$$L = \frac{(4\pi \times 10^{-7} \frac{\text{V-sec}}{\text{amp-m}}) (100^2) (40 \text{ cm}^2)}{0.1 \text{ m}}$$

100 cm = 1 m      100 cm x 100cm = 1 m<sup>2</sup>

$$\frac{10,000 \text{ cm}^2}{40 \text{ cm}^2} = X \text{ m}^2$$

$$X = 4 \times 10^3 \text{ m}^2$$

$$L = \frac{(4\pi \times 10^{-7} \frac{\text{V-sec}}{\text{amp-m}}) (10^4) (4 \times 10^3 \text{ m}^2)}{0.1 \text{ m}}$$

$$L = 50.24 \times 10^{-4} \frac{\text{V-sec}}{\text{amp}}$$

$$L = 5.0 \times 10^{-4} \frac{\text{V-sec}}{\text{amp}} = 5.0 \times 10^{-4} \text{ H}$$

**Introduction to Question 8**

An electric motor consists of an armature surrounded by a strong permanent magnet. The armature is many loops of wire wrapped around a heavy piece of metal. When the motor is turned on, alternating current flowing through the wire loops in the armature generates a magnetic field which alternately attracts and repels the magnetic field in the permanent magnet around the armature. The alternating magnetic force causes the armature to rotate.

slow speed: low back EMF       armature

high speed: high back EMF

heat = energy (joules)

power (watts) = joules/sec

power x time = energy (joules, kilowatt-hours)

power = IV

$$I \left( \frac{\text{coulombs}}{\text{sec}} \right) \times V \left( \frac{\text{joules}}{\text{coulomb}} \right) = \text{joules/sec}$$

$$V = IR$$

power = I<sup>2</sup>R (joules/sec, watts)

As the wire loops begin to rotate and cut through the magnetic field lines of the permanent magnet, the wire loops induce their own voltage. By the right hand rule, this induced voltage opposes the voltage, and the current, that started the armature rotating in the first place, as predicted by Lenz' law. This induced voltage is called a "reverse voltage" or "back-emf."

Until the back EMF kicks in, however, the current flowing through the armature is very high, so high, in fact, that the current supplying other electrical items on that circuit actually drops. The lights may dim, for example.

Once the armature picks up speed, the back EMF begins to rise. This lowers the current through the armature, allowing current through the rest of the circuit to return to normal.

The back EMF can sometimes be visualized when a motor is suddenly unplugged. The armature continues to spin and create a back EMF, but with no forward voltage to oppose the back EMF, the large voltage of the back EMF is able to produce a spark in the motor.

If, for some reason, the armature slows down, the back EMF will drop, which allows lots of current to flow through the motor. If, for example, the motor is suddenly forced to turn its axle against a heavy load, the axle and the armature will slow down.

The EMF drops and this allows more current to flow through the motor and provide more power to the motor. However, allowed to continue, the high current can generate enough heat to damage the wires inside the motor.

The reason high current raises the temperature of wires is that the heat generated by a current correlates with the current squared times the resistance of the wire,  $I^2$  times  $R$ .

Heat is energy, and energy is measured in joules. Joules generated per second is the definition of power, and its units are watts.

Power times time, then, is energy in joules or kilowatt-hours. Power is current times voltage,  $I$  times  $V$ . Since voltage is  $I$  times  $R$ , power is  $I$  squared times  $R$ .

It doesn't take much of an increase in current before the wire is generating many joules of heat.

8. A motor draws 60 amps of current when it is first turned on using 240 volts. The motor generates a back EMF of 100 volts as it reaches operating speed. How much current does it use at operating speed?

- (A) 30 A
- (B) 35 A**
- (C) 40 A
- (D) 45 A
- (E) 50 A

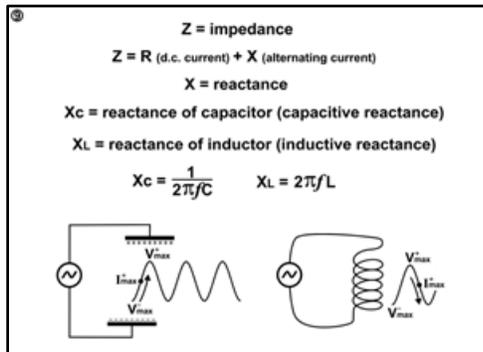
When first turned on, the motor draws a current of 60 amps from the 240 volts, meaning its resistance is 4 ohms.

At operating speed, the back voltage of 100 volts lowers the incoming voltage to 140 volts. Because the resistance of the motor doesn't change, its current at full operating speed drops to 35 amps.

$I = 60 \text{ A}$
$V = 240 \text{ V}$
$V = IR$
$240 \text{ V} = 60 \text{ A} \times R$
$R = 4 \text{ ohms}$
back EMF = 100 V
$240 \text{ V} - 100 \text{ V} = 140 \text{ V}$
$140 \text{ V} = I \times 4 \text{ ohms}$
$I = 35 \text{ A}$

### Introduction to Question 9

The resistance to the flow of alternating current is called impedance, symbolized by the letter  $Z$ . Impedance is made up of two components: resistance to the steady flow of current, and resistance to any rapid change in current. Resistance to the steady flow of current is called resistance,  $R$ . Resistance to changes in current is called reactance.



Reactance refers to the fact that the resistance is a reaction to any change in current. Capacitors and inductors both offer reactance to alternating current.

The units for reactance are ohms. The symbol for reactance is X. The reactance by a capacitor is called capacitive reactance. The reactance by an inductor is called inductive reactance.  $X_C$  is the symbol for capacitive reactance, and  $X_L$  is the symbol for inductive reactance.

The ability of a capacitor and an inductor to resist alternating current depends on the frequency of the alternating current and on the capacitance of the capacitor or inductance of the inductor.

Capacitive reactance is measured by the formula:

$$X_C = 1/2\pi f C$$

This formula says that a capacitor resists alternating current less when the alternating current is at high frequency and less when the capacitance of the capacitor is high. C stands for capacitance in units of farads.

The reason the capacitive reactance is less for a large capacitance capacitor facing high frequency alternating current is that the more electrical charges there are on the capacitor plates, the easier it is for a small percentage of them to move back and forth, and the high frequency means they don't have to move very far before reversing direction.

An inductor does the opposite. At high frequency, an inductor resists rapid changes in voltage, according to the formula  $X_L = 2\pi f L$ . This formula also says that the higher the inductance, the better the inductor is at resisting the back and forth changes in voltage. L stands for inductance in units of henries.

Each voltage flip in an inductor is met with an induced back voltage resisting the sudden change in current. The more often the current flips, the more time the inductor spends resisting the changing current.

In a capacitor and inductor, then, both the current and the voltage change with each cycle, but they do so at different times.

In a capacitor, the current surges onto the capacitor plates and then slows to a halt as the electrical charges accumulate on the capacitor plate.

The maximal flow of current occurs before the maximal voltage occurs on the capacitor plate. It takes a quarter of a cycle, or 90 degrees, for the voltage to reach its peak after the current reaches its peak.

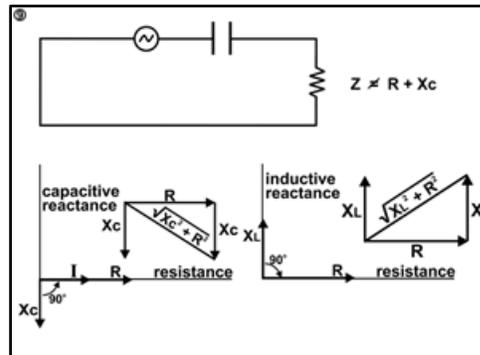
In an inductor, on the other hand, a rising voltage causes the current to increase, but the current takes a quarter of a cycle to respond.

The reason the current lags behind the voltage in an inductor is that the current cannot change direction as quickly as the voltage can.

Resistors resist the flow of steady current, while capacitors and inductors react to the changing flow of alternating current. When resistors and capacitors, or resistors and inductors, are both present in an alternating current circuit, their resistance and reactance combine to form impedance to the flow of alternating current.

9. Cont.

Here is a circuit composed of a capacitor and a resistor. The impedance of this circuit is the resistance of the resistor,  $R$ , and the reactance of the capacitor.



Impedance is not simply  $R$  plus  $X_c$ , because the resistance and the reactance are out of phase with each other.

If resistance and reactance are represented by vectors, being out of phase with each other means their vectors are pointing in different directions, and they have to be added like vectors.

This vector diagram represents the resistor's resistance along the X axis and the capacitor's reactance along the Y axis. A vector at zero degrees lies along the X axis. The vector rotates counterclockwise as it rotates 360 degrees.

The current starts out along the X axis at zero degrees. The resistance vector also lies along the X axis, because when the current started out, the resistance changed immediately.  $I$  and  $R$  are exactly in phase with each other, so  $R$  points in the same direction as  $I$ .

The capacitor's reactance is aimed due south, 90 degrees behind the current and the resistance vectors, because it reacted to the change in current with a 90 degree lag.

The opposite situation occurs in an inductor. The inductor’s reactance precedes the change in current, so the inductor’s reactance vector is aimed due north, ahead of the current and the resistance vectors.

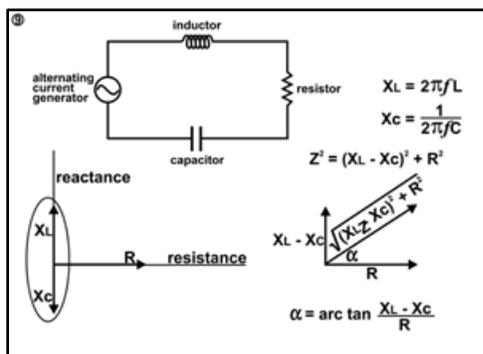
Resistance and reactance together equal impedance. To combine two vectors at right angles to each other, the tail of one vector is placed at the tip of the other vector to form the hypotenuse of a right triangle.

For a resistor and capacitor circuit, impedance becomes the square root of R squared plus Xc squared.

For a resistor and inductor circuit, impedance becomes the square root of R squared plus XL squared.

9. Cont.

What do you predict the impedance would be for a circuit containing a resistor and both a capacitor and an inductor?



In that case, reactance would be  $X_L$  minus  $X_c$ .

Impedance would be the hypotenuse of a right triangle with  $X_L$  minus  $X_c$  on one side and  $R$  on the other.

$Z$  squared would equal  $X_L$  minus  $X_c$ , squared, plus  $R$  squared, with the mathematic definitions of  $X_L$  and  $X_c$  shown above.

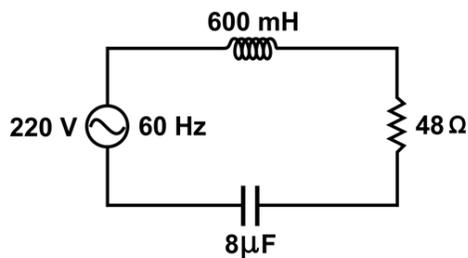
Impedance would then be the square root of  $X_L$  minus  $X_c$ , squared, plus  $R$  squared.

The angle between the resistance vector and the impedance vector is the phase angle, alpha. The phase angle is the arc tangent of the inductive reactance minus the capacitive reactance,  $X_L$  minus  $X_c$ , over the resistance.

When the phase angle is positive, meaning the inductor’s reactance is greater than the capacitor’s reactance, then the circuit acts like a resistor-inductor circuit, and the current lags behind the voltage.

When the phase angle is negative, the circuit acts like a resistor-capacitor circuit, and the voltage lags behind the current.

9. In this circuit, the capacitor is 8 microfarads, the inductor 600 millihenries, the resistor 48 ohms, and the voltage generated by the 60 cycle generator is 200 volts. What is the current and what is the phase angle?



- (A) 2.1 A, -46.8°
- (B) 1.8 A, -53.4°
- (C) 2.3 A, -57.3°
- (D) 2.0 A, -61.4°
- (E) 1.9 A, -65.6°**

In order to determine the phase angle, we need to calculate the resistance and the reactance, because the phase angle will be the arc tangent of the reactance over the resistance.

The resistance is voltage over current. The voltage is 220 volts. The current is the 220 volts divided by the total impedance of the circuit.

The total impedance is the square root of  $X_L$  minus  $X_C$ , squared, plus  $R$  squared.

Capacitive reactance,  $X_C$ , is 1 over  $2\pi$  times the frequency of the alternating current times the capacitance of the capacitor.

$X_C$  works out to be 332 ohms.

Inductive reactance,  $X_L$ , is  $2\pi f$  times the inductance,  $L$ , in henries.

$X_L$  works out to be 226 ohms.

Total impedance,  $Z$ , the, is the square root of 226 minus 332, squared, plus 48, squared.

$Z$  equals 116 ohms.

Current is voltage divided by total impedance,  $Z$ .

220 volts divided by 116 ohms is a current of 1.90 amps.

The phase angle, alpha, is the arc tangent of the inductive reactance,  $X_L$ , minus the capacitive reactance,  $X_C$ , over the resistance of 48 ohms.

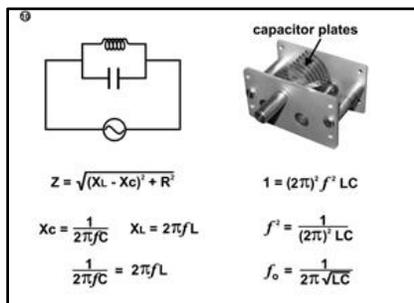
$X_L$  minus  $X_C$  is minus 106 ohms, and the arc tangent of minus 106 over 48 is minus 65.6 degrees.

The negative sign means that  $X_C$  is greater than  $X_L$ , so this combination of inductor and capacitor acts more like a capacitor with the current leading the voltage by 65.6 degrees.

$X_C = \frac{1}{(6.28)(60)(8 \times 10^{-6})}$   
 $X_C = 332 \text{ ohms}$   
 $X_L = (6.28)(60)(0.6)$   
 $X_L = 226 \text{ ohms}$   
 $Z = \sqrt{(226 - 332)^2 + 48^2}$   
 $Z = \sqrt{13,540}$   
 $Z = 116 \text{ ohms}$   
 $I = \frac{V}{Z}$   
 $I = \frac{220 \text{ volts}}{116 \text{ ohms}} = 1.9 \text{ A}$   
 $\text{arc tan } \frac{X_L - X_C}{R}$   
 $\text{arc tan } \frac{-106}{48} = -2.208$   
 $\alpha = -65.6^\circ$

**Introduction to Question 10**

Because capacitors are better at blocking low frequency alternating current and inductors are better at blocking high frequency alternating current, when attached together in parallel, a capacitor and inductor only allow medium frequency alternating current to pass. In fact, depending on the specific capacitance and inductance, one frequency in particular meets the least impedance and is allowed to pass with peak efficiency. That frequency is called the resonant frequency.



capacitor plates

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$1 = (2\pi)^2 f^2 LC$$

$$X_C = \frac{1}{2\pi fC} \quad X_L = 2\pi fL \quad f^2 = \frac{1}{(2\pi)^2 LC}$$

$$\frac{1}{2\pi fC} = 2\pi fL \quad f_o = \frac{1}{2\pi\sqrt{LC}}$$

Since impedance, Z, is the square root of  $X_L$  minus  $X_C$ , squared, plus R squared, impedance is least when  $X_L$  minus  $X_C$  squared is zero. That occurs when  $X_L$  equals  $X_C$ .

When  $X_L$  equals  $X_C$ , the resonant frequency is 1 over 2 pi times the square root of the inductance times the capacitance.

Turning the station dial on a radio rotates the capacitor plates and changes the capacitance of the capacitor. This changes the radio's resonant frequency. When the resonant frequency matches the frequency of the radio station, the signal becomes clear.

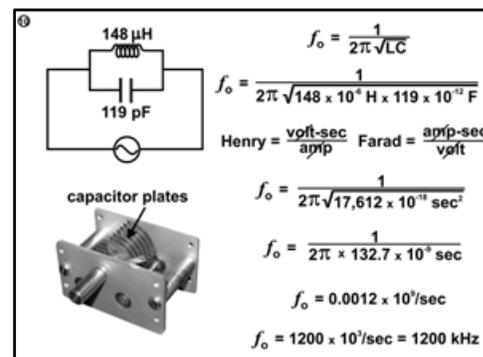
10. If a radio circuit contains a 148  $\mu\text{H}$  inductor in parallel with a variable capacitor, and when the capacitor is adjusted to 119 pico Farads, the radio station becomes clear, what frequency is the radio station broadcasting at?

- (A) 1000 kHz
- (B) 1100 kHz
- (C) 1200 kHz**
- (D) 1300 kHz
- (E) 1400 kHz

The resonant frequency is 1 over 2 pi times the square root of the inductance times the conductance.

In this case, the resonant frequency is 1 over  $2\pi$  times the square root of 148 times  $10^{-6}$  henries times 119 times  $10^{-12}$  farads.

Henries are volt-seconds per amp and farads are amp-seconds per volt. The resonant frequency works out be 1200 kilo Hz. The radio station is broadcasting at 1200 kilo Hz.



capacitor plates

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi\sqrt{148 \times 10^{-6} \text{ H} \times 119 \times 10^{-12} \text{ F}}}$$

Henry =  $\frac{\text{volt-sec}}{\text{amp}}$  Farad =  $\frac{\text{amp-sec}}{\text{volt}}$

$$f_o = \frac{1}{2\pi\sqrt{17,612 \times 10^{-18} \text{ sec}^2}}$$

$$f_o = \frac{1}{2\pi \times 132.7 \times 10^{-9} \text{ sec}}$$

$$f_o = 0.0012 \times 10^9/\text{sec}$$

$$f_o = 1200 \times 10^3/\text{sec} = 1200 \text{ kHz}$$