1. For flat mirrors, the angle of incidence equals the angle of reflection.

This 6 foot gentleman wants to buy a mirror that allows him to see his entire body. What is the smallest mirror he can buy, where should the top of the mirror be hung, and how far away from him should it be hung?

(A) The minimum length of the mirror is ¾ of his height. The top of the mirror should be even with the top of his head. The mirror should be 3 feet in front of him.
(B) The minimum length of the mirror is ½ his height. The top of the mirror should be even with the top of his head. The mirror should be 3 feet in front of him.
(C) The minimum length of the mirror is ½ the distance from his eyes to his feet. The top of the mirror should be even with the middle of his forehead. The mirror should be 3 feet in front of him.
(D) The minimum length of the mirror is ½ the distance from his eyes to his feet. The top of the mirror should be even with the middle of his forehead. The mirror can be any distance in front of him.
(E) The minimum length of the mirror is ½ his height. The top of the mirror should be even with his eyes. The mirror should be 3 feet in front of him.

Because the angle of incidence with the mirror equals the angle of reflection, the man's visual path is an isosceles triangle on its side.

For his visual path to reach his feet, the bottom of the mirror only has to be half the distance from his eyes to his feet.

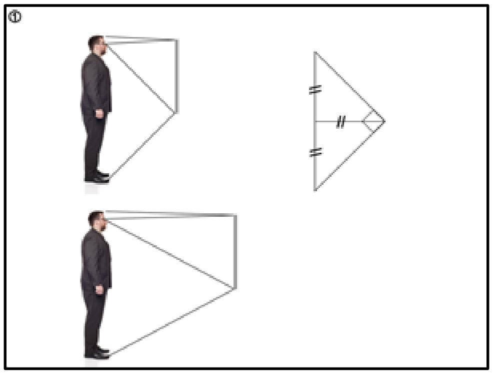
The reasoning holds for seeing the top of his head. The top of the mirror only has to be halfway between his eyes and the top of his head.

It makes no difference how far away the mirror is. So long as the mirror stretches from his mid-forehead to half the distance between his eyes and his feet, he will always be able to see his whole body in the mirror.

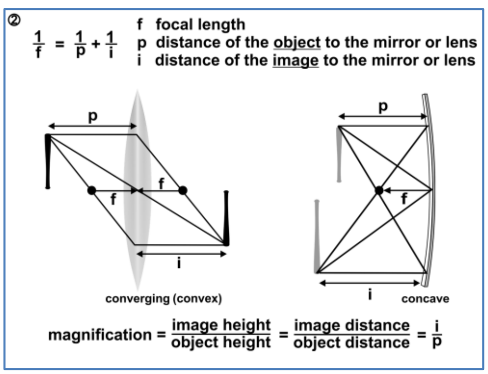
How far away does the mirror have to be for the visual pathway to form a right angle?

If the visual pathway forms a right angle with the mirror, the right angle is bisected into two 45 degree angles, then the upper and lower triangles both form isosceles right triangles.

The horizontal distance to the mirror is now the same as half the distance between the eyes and feet.



Introduction to Question 2



The lens formula is $\frac{1}{f} = \frac{1}{p} + \frac{1}{i}$. One over the focal length equals one over the object distance plus 1 over the image distance. The lens formula applies to both mirrors and lenses.

f is the focal length of the mirror or lens. f is negative for a diverging lens or a convex mirror. For spherical mirrors and lenses, f is one-half the radius of curvature.

p measures how far the object is from the mirror or lens. Since p is always in front of the mirror or lens, p is always positive.

i measures how far the image forms from the mirror or lens. A positive i means the image forms where, so to speak, it should form -- in front of the mirror or behind a lens.

A negative i means the image forms where it shouldn't, namely, behind the mirror or in front of the lens. Images that form behind the mirror or in front of the lens are called "virtual" images. Virtual images cannot be projected onto a screen.

Only a convex lens can form a real image. A concave lens always forms virtual images.

Magnification is the ratio of image height over object height, which is the same ration is image distance over object distance, i over p.

To determine the image's orientation, whether it's upright or inverted, recall that for converging lenses and concave mirrors, images are always inverted until the object is brought inside the focal point, and then the image becomes upright.

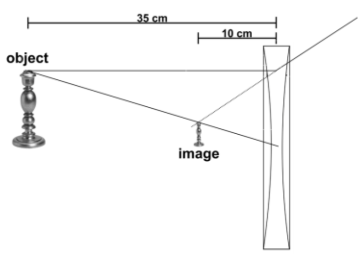
For diverging lenses and convex mirrors, the image is always upright.

Objects and their images can be graphed out using a number of different rays. One ray runs horizontally from the top of the object to the mirror or lens and then through the focal point.

Another ray passes from the top of the object through the focal point to the mirror or lens and then horizontally from there. For mirrors , this ray is slightly off because of the curvature of the mirror, a problem called spherical aberration.

A third ray passes from the top of the object straight through the center of the lens without deflecting at all. For mirrors, the ray passes from the top of the object to the center of the mirror and reflects back at the same angle.

2. What focal length lens is needed to form a virtual image 10 cm in front of the lens when the object is placed 35 cm in front of the lens?



(A) a converging lens with a focal length of 14 cm

(B) a diverging lens with a focal length of 14 cm

(C) a converging lens with a focal length of 25 cm

(D) a diverging lens with a focal length of 25 cm

(E) a diverging lens with a focal length of 10 cm

Because the image forms in front of the lens, i is -10 cm.

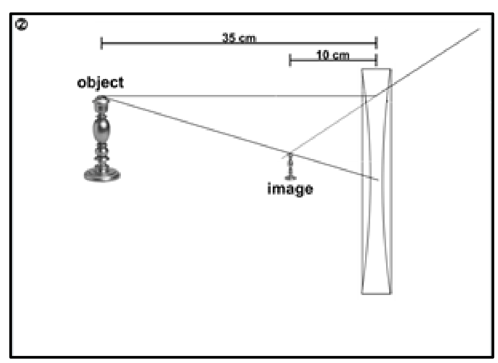
Plugging in the values for the distance to the object and the distance to the image, we get:

$\frac{1}{f} = \frac{1}{35} + \frac{1}{-10}$, which is $\frac{1}{f} = \frac{1}{35} - \frac{1}{10}$. This works out to be $\frac{1}{f} = \frac{1}{-14}$.

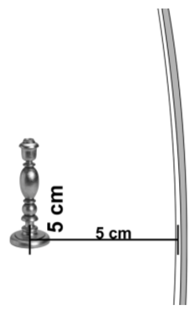
F = -14

Because the focal length is negative, the lens has to be a diverging lens.

By passing a horizontal line from the top of the object to the lens and another line through the center of the lens, the intersection of the two lines locates the top of the image.



3. A 5 cm object is placed 5 cm in front of a concave spherical mirror with a focal length of 10 cm. Where does the image form, how high is it, and is it upright or upside down?



(A) 10 cm tall, 10 cm behind the mirror, and upright
(B) 10 cm tall, 10 cm behind the mirror, and upside down
(C) 5 cm tall, 5 cm behind the mirror, and upright
(D) 5 cm tall, 5 cm behind the mirror, and upside down
(E) 10 cm tall, 10 cm in front of the mirror, and upright

The lens formula, $\frac{1}{f} = \frac{1}{p} + \frac{1}{i}$, applies to both mirrors and lenses.

$\frac{1}{10} = \frac{1}{5} + \frac{1}{i}$.

This works out to $-\frac{1}{10} = \frac{1}{i}$.
i = -10

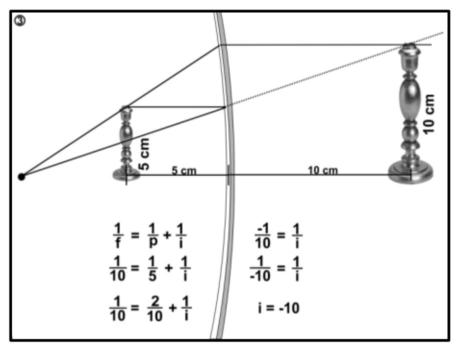
Because i is negative, the image is where it shouldn't be, namely behind the mirror, 10 cm behind the mirror.

The magnification of an image is the ratio of the image distance divided by the object distance, or $\frac{1}{p}$.

In this case, since the image forms twice as far from the mirror as it started out, it must be twice as high, or 10 cm tall.

All images reflected off a concave mirror are inverted except when the object is moved inside the focal point, in which case the image becomes upright.

The correct answer is a.



4. The following problem demonstrates the problem of spherical aberration.

What is the magnification of a concave mirror with a focal length of 25 cm when a 10 cm object is placed 20 cm in front of the mirror?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

Using the mirror equation, $\frac{1}{f} = \frac{1}{p} + \frac{1}{i}$, and filling in the values for f and p, we get:

$\frac{1}{25} = \frac{1}{20} + \frac{1}{i}$. This works out to be $\frac{4}{100} - \frac{5}{100} + \frac{1}{i}$.

$24 - \frac{1}{100} = \frac{1}{i}$, and i = -100

The image forms 100 cm from the mirror. The negative sign indicates that that the image forms behind the mirror.

Image magnification corresponds to the ratio of the image distance divided by the object distance. $\frac{i}{p}$ in this case is $\frac{100}{120}$, or 5. The image is 5 times larger than the object.

Because the object was placed inside the focal point, the image is upright.

We can confirm this by drawing the light rays. We'll begin with the mirror. The mirror has the spherical shape of a circle with a radius twice that of the focal length. Since the focal length is 25 cm, the radius is 50 cm.

Here is 50 cm radius circle. The focal point is 25 cm away from the circumference. The object is 5 cm closer to the circumference, 20 cm away from the circumference.

When a ray is extended horizontally from the top of the object to the mirror, the ray reflects back through the focal point.

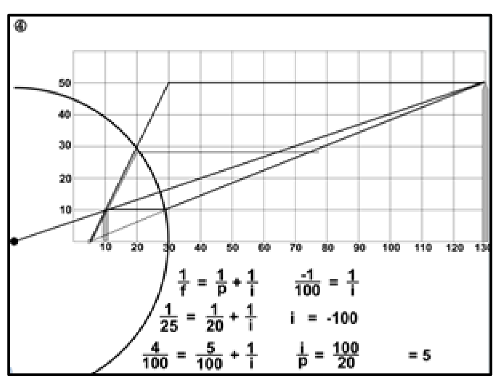
Extending that ray will eventually allow us to determine where the image forms.

A second ray extending from the center of curvature intersects the circumference perpendicularly, and reflects back on itself.

Extending this ray allows it to intersect the first ray at a point 100 meters beyond the mirror. The image that forms here is 5 times taller than the original object.

Using our third ray fails to intersect where the first two did. Passing a ray from the focal point through the object should cause the ray to reflect back horizontally from the mirror. When the horizontal ray is extended to the right, it intersects the other two rays at the wrong spot. The only way to get this third ray to intersect the other two rays is to pass the ray through the mirror to a point 50 cm high and then allow it to pass horizontally.

The failure of the third ray to locate the image is due to spherical aberration. Which means ray tracing only works when the rays make a very small angle close to the principal axis.



5. A 10 cm object 40 cm from a concave mirror with a focal length of 30 cm creates an image that's __________.

(A) 60 cm behind the mirror, 30 cm tall, upright
(B) 60 cm in front of the mirror, 15 cm tall, inverted
(C) 120 cm in front of the mirror, 15 cm tall, inverted
(D) 120 cm behind the mirror, 30 cm tall, upright
(E) 120 cm in front of the mirror, 30 cm tall, inverted

Beginning with the lens maker's formula, $\frac{1}{f} = \frac{1}{p} + \frac{1}{i}$, we get:

$$\frac{1}{30} = \frac{1}{40} + \frac{1}{i}$$

i works out to be 120 cm.

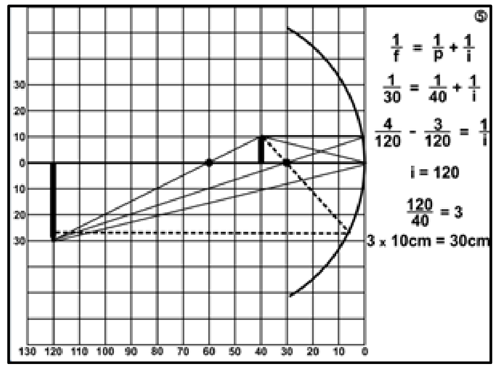
Because i is positive, the image forms where it should form, in front of the mirror.

It's magnification is $\frac{i}{p}$, or $\frac{120}{40}$. The image is three times larger than the object. Since the object is 10 cm tall, the image is 30 cm tall.

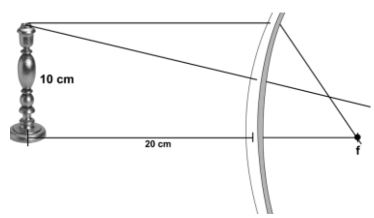
Because the object is outside the focal point, the image is inverted.

A graphical representation of this problem confirms the location of the image.

The last ray again demonstrates spherical aberration.



6. A 10 cm object is placed 20 cm in front of a convex mirror with a focal length of 10 cm. Where does the image form, how tall is it, and is it upright or upside down?



(A) $\frac{10}{3}$ cm tall in front of the mirror and upright

(B) $\frac{10}{3}$ cm tall in front of the mirror and upside down

(C) $\frac{10}{3}$ cm tall behind the mirror and upright

(D) $\frac{10}{3}$ m tall behind the mirror and upside down

(E) $\frac{20}{3}$ cm tall in front of the mirror and upright

The same lens formula is used. In a convex lens, the focal point is behind the mirror, so the focal length is negative.

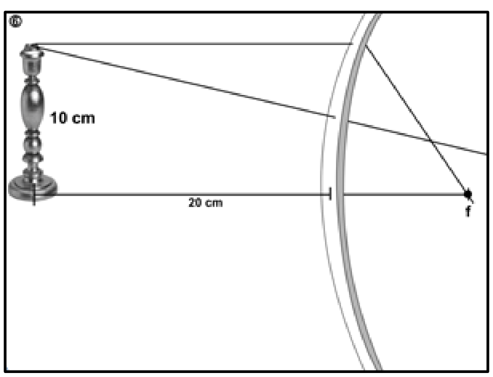
Filling in the numbers for f, p, and i, i turns out to be $\frac{-20}{3}$.

A positive i means the image formed in front of the mirror where you would expect it to form. Because i is negative, the image forms $\frac{20}{3}$ cm behind the mirror.

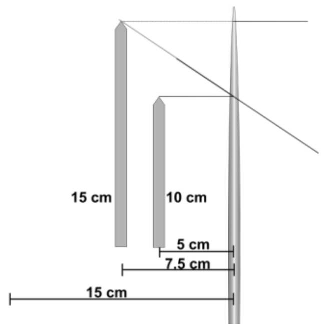
The magnification of the object is i over p, or -$\frac{-20}{3}$ divided by 20, or $-\frac{1}{3}$. The magnification is one-third, so the image is $\frac{10}{3}$ cm tall.

Because the mirror is convex, the image is upright. The answer is C.

This answer is confirmed by drawing a line from the top of the object horizontally to the mirror, and then down to the focal point. The other line is drawn from the top of the object to the center of curvature for the mirror, which is twice the distance of the focal point. The image forms where the two lines intersect.



7. A 10 cm object is placed 5 cm in front of a converging lens with a focal length of 15 cm. Where does the image form, how tall is the image, and is it upright or upside down?



(A) 10 cm tall in front of the mirror and upright
(B) 10 cm tall in front of the mirror and upside down
(C) 15 cm tall behind the mirror and upright
(D) 15 cm tall behind the mirror and upside down
(E) 15 cm tall in front of the mirror and upright

The lens formula also works for lenses.

1 over the focal length equals 1 over the object distance plus 1 over the image distance. For converging lenses, the focal length is positive. For diverging lenses, the focal length is negative.

When the values for f and p are inserted, i works out to be minus 7.5.

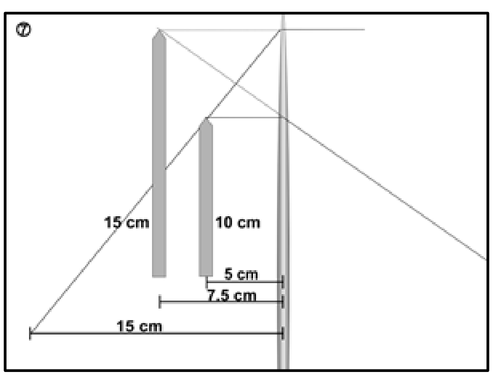
When i is positive, it means the image forms where it should form – behind the lens. When i is negative, it means the image forms in front of the lens, in this case 7.5 cm in front of the lens. But what size?

Magnification is i over p, which in this case is 1.5. The image is magnified one and half times, or 15 cm. Because the object was placed inside the focal point, the image is upright. The answer is e.

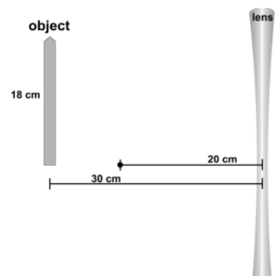
59This answer is confirmed by drawing a line from the focal point through the top of the object to the lens, and extended horizontally in both directions.

The other line is drawn from the top of the object horizontally to the lens and then down to the focal point on the other side of the lens. This line extends to the top of the image.

An object held closer than the focal length is how a magnifying glass works.



8. Here is a diverging lens with a focal length of -- because it's a diverging lens -20 cm. An 18 cm tall object is placed 30 cm in front of the lens. Where does the image form, how tall is the image, and is it upright or upside down?



(A) 12 cm tall, 7.2 cm in front of the mirror and upright
(B) 12 cm tall, 7.2 cm in front of the mirror and upside down
(C) 7.2 cm tall, 12 cm in front of the mirror and upright
(D) 7.2 cm tall, 12 cm behind the mirror and upright
(E) 7.2 cm tall, 12 cm behind the mirror and upside down

The lens formula says that 1 over the focal length equals 1 over the object distance plus 1 over the image distance.

When i is positive, it means the image forms where it should form – behind the lens. When i is negative, it means the image forms in front of the lens.

P is 30 cm and f is -20 cm. Inserting these values into the lens formula gives a value for i of -12 cm.

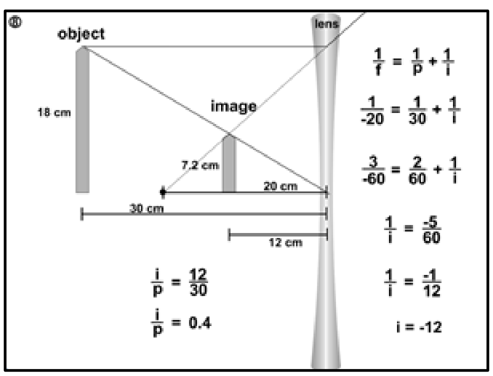
The negative sign for the image indicates that the image is on the wrong side of the lens, meaning in front of the lens.

It's height is the magnification of $\frac{i}{p}$. $\frac{i}{p}$ is $\frac{12}{30}$, or 0.4.

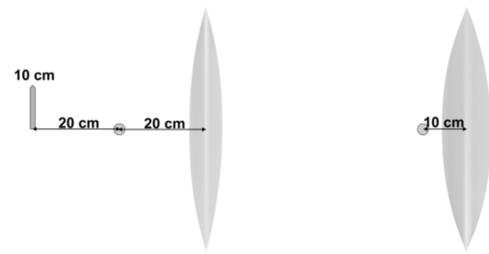
40 percent of 18 cm is 7.2 cm.

Because the lens is a diverging lens, the image is upright.

The answer is c.



9. Here are two lenses 60 cm apart. The first lens has a focal length of 20 cm and the second lens, 10 cm. A 10 cm object is placed 40 cm to the left of the first lens. Where does the image form, how high is it, and is it upright or upside down?



(A) 10 cm to the right of the second lens, 10 cm tall, upright
(B) 10 cm to the right of the second lens, 10 cm tall, upside down
(C) 20 cm to the right of the second lens, 10 cm tall, upside down
(D) 20 cm to the right of the second lens, 10 cm tall, upright
(E) 20 cm to the right of the second lens, 20 cm tall, upright

Starting with the lens formula, $\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$,

$\frac{1}{20} = \frac{1}{48} + \frac{1}{i}$

i = 40 cm.

Because i is positive, the image forms in back of the lens.

Its magnification is $\frac{i}{p}$, or $\frac{40}{40}$, so the image is still 10 cm tall.

Because the object was placed outside the focal point, the image is upside down.

This 10 cm tall image is now 20 cm to the left of the second lens, so,

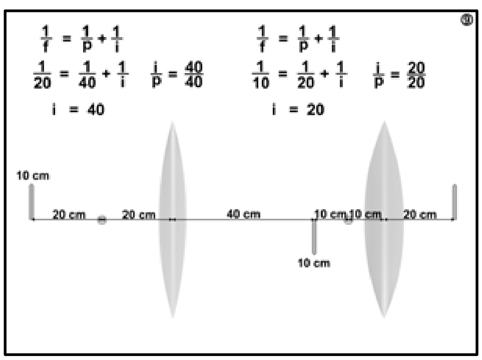
$\frac{1}{10} = \frac{1}{20} + \frac{1}{i}$

i = 20

Because i is positive, the second image is 20 cm to the right of the second lens.

Its magnification is $\frac{20}{20}$, so the image is still 10 cm tall.

Because the object was outside the focal point, the image is upside down from its starting position, and is now upright. The answer is d.



10. In 1801, Thomas Young showed that he could determine the wavelength of light by passing a beam of light through two slits and recording their interference pattern at a distance L from the slits.

All he had to do was measure the distance between the two slits and the distance from the central interference band to the one next to it.

So if Young placed the slits 0.4 mm apart, and L was 1 meter, and the first interference band 2.5 mm from the midline, what was the wavelength of the light in nanometers?

(A) 525 nm
(B) 750 nm
(C) 1000 nm
(D) 1250 nm
(E) 1450 nm

Here is Young's experiment, which enabled him to arrive at a formula for wavelength, lambda (λ).

Young said that the first interference band formed where it did because when the light passed through the two slits, the light passing through the lower slit had a longer path to travel than light passing through the upper slit. The longer length was λ, exactly one whole wavelength.

The second thing he said was that the two gray triangles were similar, one formed by sides λ and d, and the other by Y and L. Because they are similar, the ratio of Y to L is equal to the ratio of λ to d. As you can see, he fudged this a little because d is the hypotenuse of the small triangle but L is not. Because of how long L was and how short Y was, Young felt that this minor fudging would still give an accurate measurement of λ.

If $\frac{Y}{L} = \frac{\lambda}{d}$, then, wavelength, λ, equals d, the distance between the two slits, times Y, the distance from the midline to the first interference band, divided by L, the distance separating the slits from the interference pattern.

If d equals 0.4 mm and L equals 1 meter, and Y equals 2.5 mm, we can solve for λ after first converting meters and millimeters to centimeters.

λ = d, 0.04 cm, times Y, 0.25 cm, divided by L (100 cm. λ = $\frac{0.04 cm\ x\ 0.25\ cm}{100\ cm}$).

λ = 10^{-4} cm.

By multiplying 10^{-4} cm by fractions equal to 1, 10^{-4} cm = 1000 nm.

1000 nm wavelength electromagnetic radiation is infrared radiation.